Against the impossible picture: Feynman's heuristics in his search for a divergence-free quantum electrodynamics

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Abstract

Arguably, the development of Feynman diagrams not only resulted in a useful tool for calculations but also brought about deep conceptual changes in the theory of quantum electrodynamics. Starting from this thesis, I try to bring to the fore a particular aspect of it. I maintain that the function of Feynman diagrams is not exhausted by their use in the application of the finished theory to concrete cases. Rather, Feynman diagrams are one of the results of Feynman's more general search for appropriate means of representation. Accordingly, the development of Feynman diagrams is a characteristic example of Feynman's heuristics.

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1 Introduction

Richard Phillips Feynman was born in the state of New York in 1918 and died in Los Angeles in 1988. In 1942 he was awarded his doctoral degree and, in 1948, he published a condensed and revised version of his thesis. In 1965, he won the Nobel prize in physics together with Sin-Itiro Tomonaga and Julian Schwinger "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles".[1] One of Feynman's most important contributions to the research in this domain is his article "Space-time approach to quantum electrodynamics" published in 1949 (Feynman 1949). In this article, he published the first of the diagrams which later were given his name. Freeman Dyson, one of his close colleagues, used the diagrams in a publication even before Feynman himself. Feynman diagrams have been intensively used and further developed up to the present day by theoretical particle physicists as well as physicists working in other fields, in particularly those working in condensed matter physics.[2] Feynman diagrams are incontestably one of the most useful tools for organizing perturbative calculations in the aforementioned research fields.

In *The Genesis of Feynman Diagrams* (Wüthrich 2010), I intended to show how the development of Feynman diagrams not only resulted in a useful tool for calculations but also brought about deep conceptual changes in the theory of quantum electrodynamics. In the following, I try to bring to the fore a particular aspect of this conclusion. I maintain that the function of Feynman diagrams is not exhausted by their use in the application of the finished theory to concrete cases. Rather, Feynman diagrams are one of the results of Feynman's more general search for appropriate representations. As such, they play an important part in his attempt to find a divergence-free quantum electrodynamics and are characteristic for Feynman's heuristics.

Drawing on my more detailed account (Wüthrich 2010), I will first show how and why Feynman tries to quantize theories without a Hamiltonian function (Section 2). I will then explain how these attempts lead him to a physical interpretation of the Dirac equation through the model of the quivering electron (Section 3). I will conclude by highlighting that Feynman, whenever he faces a theoretical problem, he often tries to find an appropriate reformulation of it (Section 4), which would render the solution of the problem almost self-evident.

2 Quantizing the Wheeler–Feynman theory

In the early 1940s, when Feynman was a graduate student, one of the most pressing problems facing theoretical physicists of the time was the fact that infinite and, therefore, uninterpretable quantities arose from some of the principles of electrodynamics—in both classical electrodynamics as well as in the early attempts to establish a quantum theory.

In classical electrodynamics, the difficulties of divergences had been known for some time, and it had been hoped that quantizing the theory would eliminate them. The other strategy was that one should first remove the infinite quantities in the classical theory before one should even attempt to quantize it (see, e. g., Dirac 1938; Frenkel 1925).

[1] `http://nobelprize.org/nobel_prizes/physics/laureates/1965/`

[2] Thanks to Christian Joas for pointing out to me, on several occasions, the importance of Feynman diagrams in condensed matter physics.

It is in this context that Feynman wrote his PhD thesis, with the removal of the divergences in electrodynamics being his superordinate objective (Feynman 2005, Introduction). In his thesis, he adopts the second strategy of first trying to establish a divergence-free classical theory and then proceeding to quantize it. Indeed, together with his supervisor John Archibald Wheeler, Feynman had already developed an alternative theory of electrodynamics with the desired feature that awaited quantization.[3]

The standard procedure for quantizing a classical theory was to interpret the classical Hamiltonian function as an operator in a Hilbert space of state vectors. This operator would then determine the time evolution of the quantized system. The problem with quantizing the Wheeler–Feynman theory of electrodynamics was that it could not be formulated by specifying a Hamiltonian function (Feynman 2005, Introduction).

Feynman's solution to this problem consists in recognizing that the Hamiltonian is not the only classical quantity that can be used to determine the dynamics of a corresponding quantum system. The classical *action* can do so as well.

Using a relation between the Lagrangian of a system and the infinitesimal time evolution of quantum state vectors, which Feynman borrowed from Paul Dirac (see, e. g., Dirac 1933), Feynman was able to represent the time evolution of the quantum wave function of a system using directly the Lagrangian.

Through the iterative application of Dirac's relation Feynman sees that the wave function can be represented by a quite peculiar integral of the classical action of the system to be quantized. This classical action is usually the time integral of the Lagrangian function. However, the classical action also exists for theoretical systems which do not have a Lagrangian in the usual sense of the word, like in the Wheeler–Feynman theory. So, by using the representation of the quantum dynamics of a system by the action function of the corresponding classical system Feynman is able to generalize the quantization procedure to systems with no Lagrangian and, therefore, to systems with no Hamiltonian (Feynman 2005, p. 41).

Here we already see that the main purpose of Feynman's alternative formulation is neither to provide handy mnemonics to be used in cases which can also be described using standard formulations, nor just for the "pleasure in recognizing old things from a new point of view" (Feynman 1948, p. 367). Rather, Feynman needs the alternative formulation to be able to construct a description of important systems that cannot be described using standard formulations:

> What we have been doing so far is no more than to reexpress ordinary quantum mechanics in a somewhat different language. In the next few pages we shall require this altered language in order to de-

[3] Only a summary of a presentation of Wheeler and Feynman's theory had been published by the time Feynman started working on his thesis (see "Minutes of the Cambridge, Massachusetts, Meeting, February 21 and 22, 1941", p. 683). The published accounts of the theory are Wheeler and Feynman 1945; Wheeler and Feynman 1949.

scribe the generalization we are to make to systems without a simple Lagrangian function of coordinates and velocities. (Feynman 2005, p. 39)

3 The model of the quivering electron

After the Second World War, a condensed and revised version of Feynman's thesis was published in the *Reviews of Modern Physics* (Feynman 1948). What he described in his thesis as an iterative construction of the wave function is now interpreted as being a sum of contributions from every possible physical *path* that a particle can take.

Feynman investigates the characteristics of the paths that appear in his alternative formulation and recognizes that they are the result of a familiar physical process, namely *Brownian motion* (Feynman 1948, p. 376).

While the results of Feynman's thesis were "non-relativistic throughout" (Feynman 2005, p. 1), in the last section of the article in the *Reviews of Modern Physics*, Feynman demonstrates how one could include relativistic systems and particles with spin.

However, Feynman was not at all pleased with these treatments of spin phenomena and relativistic particles. Tantalizingly, he let the reader know that he was working on a more satisfactory treatment of these two subjects, which was not yet ready for publication:

> These results for spin and relativity are purely formal and add nothing to the understanding of these equations. There are other ways of obtaining the Dirac equation which offer some promise of giving a clearer physical interpretation to that important and beautiful equation. (Feynman 1948, p. 387)

Feynman here alludes to his attempts to reformulate the Dirac equation such that it describes explicitly the zigzagging paths familiar from Brownian motion.

In his unpublished notes[4], Feynman succeeds in obtaining the quantity that determines the evolution of the wave function in a more satisfactory manner than the "purely formal" way of the published article: In his notes, he shows how to interpret Dirac's differential equation in one dimension as the description of the model of the zigzagging electron, which, by the way, has already been described by Gregory Breit (1928) and Erwin Schrödinger (1930).

Through the model of an electron zigzagging through an infinitesimally fine space-time lattice, Feynman can now *explain* the time evolution of a relativistic electron. And, unlike in the final section of the published article, Feynman can now justify the action function, since he has derived it from a description of the zigzagging electron (Wüthrich 2010, pp. 75–77).

[4]Most of Feynman's manuscripts and letters have been collected by the Archives of the California Institute of Technology. The documents to which I refer here and in the following are reproduced in Wüthrich (2010, Chapter 4). Some of them can also be found in Schweber (1994).

4 Feynman's programme

After having successfully treated the one-dimensional Dirac equation, the next step that Feynman was to consider was the Dirac equation describing real electrons, that is, electrons moving not just in one spatial dimension but in three spatial dimensions. During his attempt to generalize his model to more than one spatial dimension, Feynman, again and again, resorts to his reformulation panacea. In a letter to his student friend Theodore Welton, Feynman writes:[5]

> Still my stuff sounds mathematical—& insofar as it is, I still don't understand it—but I will try soon to reformulate in terms of seeing how things look to someone riding with the electron.

In fact, in the same letter, Feynman is quite explicit about the strategy with which he is trying to solve the problems he is facing:

> I am engaged now in a general program of study—I want to understand (not just in a mathematical way) the ideas in all branches of theor. physics. As you know I am now struggling with the Dirac Eqn.

Feynman's aim is to describe Dirac's well-known equation in alternative ways, for he does not believe that a physical theory is completely specified by its equations. The equations have to be completed by "pictures", and several pictures are possible for the same equations:

> I find physics is a wonderful subject. We know so very much and then subsume it into so very few equations that we can say we know very little (except these equations—Eg. Dirac, Maxwell, Schrod[inger]). Then we think we have *the* physical picture with which to interpret the equations. But there are so very few equations that I have found that many physical pictures can give the same equations. So I am spending my time in study—in seeing how many new viewpoints I can take of what is known.

Feynman thus seeks what luminaries like Dirac would tell him to be impossible. This whole enterprise of devising "pictures" of quantum mechanical phenomena clashes with the usual education in quantum mechanics, which Feynman had received through, among other things, the textbook by Dirac (1935). This, however, thus not bother Feynman too much and he declares:

> I dislike all this talk of there not being a picture possible but we only need know how to go about calculating any phenomena.

Feynman knows all too well about the value of having a clear physical interpretation, a "picture", of the mathematical equations of the theory.

[5] The most important pages of Feynman's letter to Welton are reproduced in Wüthrich (2010, pp. 83–95). See also Schweber (1994, pp. 406–408).

Since the time of his PhD thesis, Feynman knows that the search for different viewpoints is not just an intellectual "pleasure"; it also has a precise goal. In the letter to Welton, he explains:

> Of course, the hope is that a slight modification of one of the pictures will straighten out some of the present troubles.

Feynman's objective is to be able to interpret the known equations in such a way that it becomes clear which assumption in the theory is causing the inconsistent conclusions in the troublesome cases. Once the culprit of the contradiction (between the theory and more general physical principles or uncontested experimental data) has been identified, it should then be possible to resolve the problem by "modifying" the problematic assumption.

Will Feynman's programme be successful? In *The Genesis of Feynman Diagrams* and, in a condensed version, also in "Feynman's struggle and Dyson's surprise revelation" (Wüthrich submitted), I try to show that, indeed, Feynman's diagrammatic method and his proposal of a divergence-free quantum electrodynamics exactly follows what he outlines in his letter to Welton. Feynman will further develop the model of a quivering electron into a model where electrons propagate in a more abstract sense of the word. He will thereby reduce the whole of quantum electrodynamic phenomena to a single fundamental process. For Feynman, this fundamental process will be the appropriate picture which reveals the problematic assumptions of the theory and suggests modifications which are able to solve the problems.

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